

Mathematics for Computer Science  
Spring 2019  
Due: 23:59, March 4, 2019

### Homework Set 1

**Textbook:** We refer to as LPV, the book by Lovasz, Pelikan and Vesztergombi, “Discrete Mathematics”. An electronic version can be found on the course website.

**Reference book:** This is a more advanced book, which we will need in the latter part of the semester. Blum, Hopcroft, Kannan, “Foundations for Data Science”. An electronic version can be found on the course website.

**Reading Assignments:** Read Chapters 1 and 2 of LPV.

**Written Assignments:** Do the following exercises from LPV:

1.8.26, 1.8.29, 2.1.8, 2.1.13, 2.5.2, 2.5.7.

**Special Problem 1** (counted as 2 exercises)(a) In the birthday problem discussed in class (with 72 students present), using the same probability space, let  $T_1$  be the event that there is a *triple-collision*, i.e. three students having the same birthday. Calculate the numerical value of  $\Pr\{T_1\}$ .

(b) In the same probability space as above, let  $T_2$  be the event that there are at least 6 disjoint *same-birthday* pairs of students. Calculate the numerical value of  $\Pr\{T_2\}$ .

**Special Problem 2** (counted as 2 exercises)(a) Develop a mathematical probability model for the Monte Hall Problem discussed in class. Specify mathematically the events corresponding, respectively, to the success of the strategies *Switch* and *No-Switch*. Show that  $\Pr\{No - Switch\} = 1/3$ , and  $\Pr\{Switch\} = 2/3$ .

(b) Suppose Host Monte Hall puts the coveted car behind doors 1, 2, 3 with probabilities  $1/2, 1/3, 1/6$  respectively (instead of uniformly as in class). Assume the contestant initially picks door 1. How should you modify your probability space? What is the value of  $\Pr\{No - Switch\}$  and  $\Pr\{Switch\}$ .

**Special Problem 3** (counted as 2 exercises) A 4 by 4 box is filled with integers  $1, 2, \dots, 15$  with the lower-right corner initially left empty. (See the figure on next page.) At any time there is exactly one cell empty. At each step you may move to the empty cell one of its adjacent number.

*Question:* Prove that the second configuration cannot be reached from the first configuration.

|    |    |    |    |
|----|----|----|----|
| 1  | 2  | 3  | 4  |
| 5  | 6  | 7  | 8  |
| 9  | 10 | 11 | 12 |
| 13 | 14 | 15 |    |

|    |    |    |    |
|----|----|----|----|
| 2  | 1  | 3  | 4  |
| 5  | 6  | 7  | 8  |
| 9  | 10 | 11 | 12 |
| 13 | 14 | 15 |    |