Mathematics for Computer Science
Spring 2019
Due: 23:59, March 11, 2019

Homework Set 2

Reading Assignments: Read Chapters 3 and 4 of the textbook *LPV* (Lovasz, Pelikan, Vesztergombi, *Discrete Mathematics*, Springer 2003.)

(*Optional Reading*:) If you are interested, read the paper (also posted on the course website) by Curtin and Warshauer (in Mathematical Intelligencer, 2006), where the history of the "cycle-following" problem (also known as the "100 Prisoners Problem" in the literature) is given and the optimality of the cycle-following strategy discussed.

Written Assignments: Do the following exercises from LPV: 3.8.8, 3.8.12, 3.8.14, 4.3.7, 4.3.14, 4.3.16.

Special Problem 1 (counted as 2 exercises) In class we considered a 3-Person Hat Problem in which each person can see the bits posted on the other two people's foreheads and tries to guess the bit on his/her own forehead, but each person is permitted to make *no* guess; the team wins if (1) no one makes an incorrect guess, and (2) at least one person makes a correct guess. We discussed a strategy for the team with a probability 3/4 to win.

Question: Give a rigorous proof that this is the best strategy possible. That is, no strategy for the team can win with probability higher than 3/4. Your model should be general enough to include strategies with randomized moves. (Recall that in class, we mentioned a particular strategy in which one person always make a random guess (0 or 1) and the other two don't speak.)

Remarks You should first define a mathematical model. Define a probability space, how any *strategy* is specified precisely, and how to define *win* as an event for the strategy. This allows you to define what the term *best strategy* means.

Special Problem 2 (counted as 3 exercises) We discussed in class a game of "Finding your IDs" for n students, using a cycle-following strategy for all the students. Let $g(n)$ be the probability of winning in this game, i.e. all students succeed in finding their ID.
(a) Give an explicit mathematical formula for $g(n)$.
(b) Determine the value of $\lim_{n\to\infty} g(n)$.
(c) Let us consider a modified game, in which each student is only allowed to search through $n/3$ boxes (instead of $n/2$). Let $h(n)$ be the probability of winning. Determine the value of $\lim_{n\to\infty} h(n)$.

Special Problem 3 (counted as 2 exercises) In a New Year's party with $2n$ people, k random names are picked to receive gifts. Assume that these $2n$ people are actually n husband-wife couples. Let $r_{n,k}$ be the probability that at least for one couple, both husband and wife win gifts.

(a) Give a mathematical formula for $r_{n,k}$.

(b) Determine the value of $\lim_{n\to\infty} r_{n,\sqrt{n}}$.